

ESCI 344 – Tropical Meteorology
Lesson 10 – Tropical Cyclones: Structure

References: *A Global View of Tropical Cyclones*, Elsberry (ed.)
Global Perspectives on Tropical Cyclones: From Science to Mitigation,
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The Hurricane, Pielke
Tropical Cyclones: Their evolution, structure, and effects, Anthes
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Reading: *An Introduction to the Meteorology and Climate of the Tropics*, Chapter 9

GENERAL

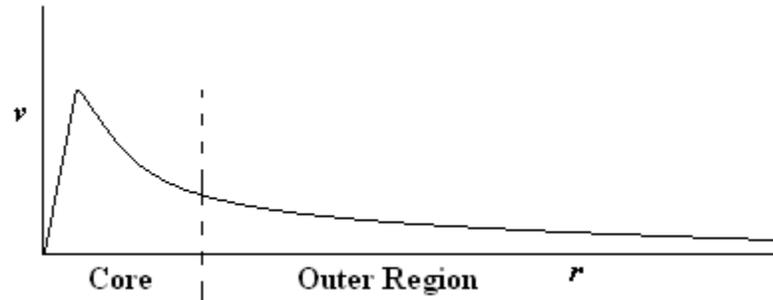
- Tropical cyclones are primarily driven by latent heating.
- As the air spirals in toward the center, it picks up latent heat and sensible heat through evaporation from the ocean.
 - Once the air is saturated, it can still pick up some sensible heat, but latent heating is the dominant mechanism.
- As the air approaches the center of the vortex, it rises in convection either in the eye wall, or in the spiral convective bands. As it rises, it cools and the water vapor condenses, giving off its latent heat.
- The warming of the air due to the latent heat release results in low-level height falls, and upper-level height rises, which helps maintain the low-level convergence of the warm-moist air.
- The air is exhausted out and away at the upper levels.

STRENGTH, SIZE, AND INTENSITY

- We will use the following definitions:
 - *Core intensity*, based on maximum winds or minimum sea-level pressure.
 - *Size*, based on the mean radius of the outermost closed isobar.
 - *Strength*, based on the shape of the outer core wind profile.
- There is great variability in the size, intensity, and strength of tropical cyclones.
 - Storms can be large and intense, small and intense, large and weak, etc.
 - You can't infer intensity based on size or strength.

DISTRIBUTION OF WIND

- The diagram below shows the typical tangential wind structure in a tropical cyclone.



- The wind structure is often represented by a modified Rankin vortex,
- where C , D , and a are empirically determined constants.

$$\begin{aligned} vr^a &= C & r > r_{\max \text{ wind}} \\ vr^{-1} &= D & r \leq r_{\max \text{ wind}} \end{aligned} \quad (1)$$

- a is usually 0.4 to 0.6.
- $a = 1$ would be a pure Rankin vortex, in which relative angular momentum is conserved.
- In cylindrical coordinates, the radial and tangential components of the momentum equation are

$$\frac{Du}{Dt} - \frac{v^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + fv + F_r \quad (2)$$

$$\frac{Dv}{Dt} + \frac{uv}{r} = -\frac{1}{\rho r} \frac{\partial p}{\partial \theta} - fu + F_\theta \quad (3)$$

where u is the radial velocity, v is the tangential velocity, r is the distance from the center of the storm, θ is the angular measure, and F_r and F_θ represent turbulent friction.

- Equations (2) and (3) are valid for both cyclonic and anticyclonic flow, as long as we use the following convention:
 - u is positive outward from the vortex
 - r is positive outward from the vortex
 - v is positive for a cyclonic vortex, and negative for an anticyclonic vortex.

- If the vortex is steady, axisymmetric, and friction is ignored, then the tangential wind is

$$\frac{v^2}{r} + fv - \frac{1}{\rho} \frac{\partial p}{\partial r} = 0, \quad (4)$$

and the vortex is in gradient balance.

- Dividing (4) by fv , we get

$$\frac{v}{fr} + 1 - \frac{1}{fv\rho} \frac{\partial p}{\partial r} = 0. \quad (5)$$

- The first term is the Rossby number, Ro .
 - In the core of the vortex, the Rossby number is large, so the Coriolis effects can be ignored. Therefore, the core of the storm is in cyclostrophic balance.
 - Outside the core, the Rossby number is of the order of unity, so gradient balance holds.
 - In either case, the wind speed depends on the pressure gradient.
- Broadly speaking, the lower the central pressure, the faster the maximum wind will be.
 - The relation between maximum winds and the central pressure is closely approximated by

$$v_{\max} = A\sqrt{p_{\infty} - p_c} \quad (6)$$

where p_{∞} is the ambient sea-level pressure outside of the circulation, and p_c is the minimum central pressure.

- The constant A is empirically determined. A value of 6.3 is often used for Atlantic hurricanes.

ABSOLUTE ANGULAR MOMENTUM

- If no energy were added or subtracted from an air parcel as it spiraled toward the center of the vortex, then its absolute angular momentum would have to be conserved.
 - The absolute angular momentum is the relative angular momentum plus the angular momentum due to the rotation of the Earth.

- An air parcel at distance r from the center of the vortex, and stationary with respect to the Earth, will have a specific absolute angular momentum (absolute angular momentum per unit mass) of

$$M_f = \omega r^2 = f_0 r^2 / 2 \quad (7)$$

just due to the rotation of the Earth.

- If the parcel has a tangential velocity v then the specific relative angular momentum is

$$M_r = vr, \quad (8)$$

so the specific absolute angular momentum is

$$M_a = M_r + M_f = vr + f_0 r^2 / 2. \quad (9)$$

- If the parcel started out at rest at a distance of 500 km, and spiraled in to a radius of 15 km, it would have attained a tangential velocity of over 600 m/s if its angular momentum were conserved.
- Obviously, parcels don't conserve angular momentum as they spiral into the center of a tropical cyclone.
- In fact, the absolute angular momentum decreases toward the center of tropical cyclones, which means that air parcels must be losing angular momentum as they spiral inward.
 - The parcels lose angular momentum through turbulent dissipation.
- The absolute vorticity of an axisymmetric vortex is

$$\eta = f_0 + \frac{\partial v}{\partial r} + \frac{v}{r}, \quad (10)$$

where the second and third terms are just the shear and curvature terms for relative vorticity. The absolute vorticity and the specific absolute angular momentum are related via

$$\eta = \frac{1}{r} \frac{\partial M_a}{\partial r}. \quad (11)$$

INERTIAL STABILITY OF A VORTEX

- A fundamental parameter for assessing how a vortex interacts with its environment is the inertial stability, which is developed mathematically below.

- In pressure coordinates the momentum equations for an axisymmetric vortex without friction are

$$\frac{Du}{Dt} = -\frac{\partial\Phi}{\partial r} + \frac{v^2}{r} + f_0v \quad (12)$$

$$\frac{Dv}{Dt} = -\frac{uv}{r} - f_0u \quad (13)$$

- If an air parcel starts out in gradient balance, but is impelled outward at a velocity u , the change in radial acceleration with time is given by taking the time derivative of (12),

$$\frac{D}{Dt}\left(\frac{Du}{Dt}\right) = -\frac{D}{Dt}\left(\frac{\partial\Phi}{\partial r}\right) + \frac{D}{Dt}\left(\frac{v^2}{r}\right) + \frac{D}{Dt}(f_0v) \quad (14)$$

- The terms on the right-hand side of (14) are evaluated as follows:

$$-\frac{D}{Dt}\left(\frac{\partial\Phi}{\partial r}\right) = -\frac{\partial}{\partial t}\left(\frac{\partial\Phi}{\partial r}\right) - u\frac{\partial}{\partial r}\left(\frac{\partial\Phi}{\partial r}\right) - \frac{v}{r}\frac{\partial}{\partial\theta}\left(\frac{\partial\Phi}{\partial r}\right) = -u\frac{\partial^2\Phi}{\partial r^2}; \quad (15)$$

$$\frac{D}{Dt}\left(\frac{v^2}{r}\right) = \frac{1}{r}\frac{Dv^2}{Dt} - \frac{v^2}{r^2}\frac{Dr}{Dt} = \frac{2v}{r}\frac{Dv}{Dt} - \frac{uv^2}{r^2} = \frac{2v}{r}\left(-\frac{uv}{r} - f_0u\right) - \frac{uv^2}{r^2} = -\frac{3uv^2}{r^2} - \frac{2f_0uv}{r} \quad (16)$$

$$\frac{D}{Dt}(f_0v) = f_0\frac{Dv}{Dt} = -f_0\frac{uv}{r} - f_0^2u \quad (17)$$

- Substituting (15) thru (17) into (14) results in

$$\frac{D}{Dt}\left(\frac{Du}{Dt}\right) = -u\frac{\partial^2\Phi}{\partial r^2} - \frac{3uv^2}{r^2} - \frac{3f_0uv}{r} - f_0^2u$$

or

$$\frac{D^2u}{Dt^2} + \left(\frac{\partial^2\Phi}{\partial r^2} + \frac{3v^2}{r^2} + \frac{3f_0v}{r} + f_0^2\right)u = 0 \quad (18)$$

- Defining

$$\omega^2 = \frac{\partial^2\Phi}{\partial r^2} + \frac{3v^2}{r^2} + \frac{3f_0v}{r} + f_0^2 \quad (19)$$

we can write (18) as

$$\frac{D^2u}{Dt^2} + \omega^2u = 0 \quad (20)$$

- Equation (20) has solution of

$$u = Ae^{i\omega t} + Be^{-i\omega t} \quad (21)$$

if $\omega^2 > 0$, in which case a parcel perturbed outward will stable oscillations around its initial radius with angular frequency ω .

If $\omega^2 < 0$ the solutions to (20) are

$$u = Ae^{\omega t} + Be^{-\omega t}, \quad (22)$$

in which case a parcel perturbed outward will accelerate a way and never return.

- We thus have established the criteria for inertial stability of a vortex:
 - An inertially stable vortex will have

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{3v^2}{r^2} + \frac{3f_0 v}{r} + f_0^2 > 0, \quad (23)$$

while an inertially unstable vortex will have

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{3v^2}{r^2} + \frac{3f_0 v}{r} + f_0^2 < 0. \quad (24)$$

- The inertial stability of a vortex is an indication of how readily the vortex can interact with its environment.
- A highly inertially stable vortex will resist any perturbations and outside influences, and will remain symmetric. A low stability vortex, on the other hand, is more readily influenced by outside disturbances, and will be less symmetric.

INERTIAL STABILITY AND ABSOLUTE VORTICITY

- The inertial stability condition, (23), can be written in terms of absolute vorticity as follows:
 - For a parcel in gradient balance, the radial geopotential gradient (pressure gradient) is

$$\frac{\partial \Phi}{\partial r} = \frac{v^2}{r} + f_0 v. \quad (25)$$

- Substituting (25) into (23) results in

$$\frac{\partial}{\partial r} \left(\frac{v^2}{r} + f_0 v \right) + \frac{3v^2}{r^2} + \frac{3f_0 v}{r} + f_0^2 > 0$$

which expands to

$$\frac{2v}{r} \frac{\partial v}{\partial r} - \frac{v^2}{r^2} + f_0 \frac{\partial v}{\partial r} + \frac{3v^2}{r^2} + \frac{3f_0 v}{r} + f_0^2 > 0$$

and through the following steps

$$\frac{2v}{r} \frac{\partial v}{\partial r} - \frac{v^2}{r^2} + f_0 \frac{\partial v}{\partial r} + \frac{2v^2}{r^2} + \frac{v^2}{r^2} + \frac{2f_0 v}{r} + \frac{f_0 v}{r} + f_0^2 > 0$$

$$\frac{2v}{r} \frac{\partial v}{\partial r} + f_0 \frac{\partial v}{\partial r} + \frac{2v^2}{r^2} + \frac{2f_0 v}{r} + \frac{f_0 v}{r} + f_0^2 > 0$$

$$\left(\frac{2v}{r} + f_0 \right) \frac{\partial v}{\partial r} + \left(\frac{2v}{r} + f_0 \right) \left(\frac{v}{r} \right) + \left(\frac{2v}{r} + f_0 \right) f_0 > 0$$

results in

$$\left(\frac{2v}{r} + f_0 \right) \left(\frac{\partial v}{\partial r} + \frac{v}{r} + f_0 \right) > 0. \quad (26)$$

- The second term in brackets is the absolute vorticity, and so we finally have the criteria for inertial stability as

$$\left(\frac{2v}{r} + f_0 \right) \eta > 0. \quad (27)$$

- Equation (27) shows that negative absolute vorticity is not a sufficient condition to have inertial instability. A vortex with negative absolute vorticity can still be inertially stable as long as

$$\left(\frac{2v}{r} + f_0 \right) < 0. \quad (28)$$

- In straight-line flow, or for weak vortexes where the curvature term in (27) can be ignored, the stability condition can be written as

$$\eta f_0 > 0. \quad (29)$$

- This is why you often hear it said that anytime absolute vorticity is negative that the flow is inertially unstable. However, keep in mind that for stronger vortexes, particularly at low latitudes, negative absolute vorticity doesn't automatically imply inertial instability.

- And finally, a reminder that the oscillation frequency of a stable vortex is given by the stability condition [recall Error! Reference source not found.], so that for a stable vortex

$$\omega^2 = \left(\frac{2\nu}{r} + f_0 \right) \eta. \quad (30)$$

- Sometimes it is helpful to write the stability condition or oscillation frequency in terms of the specific absolute angular momentum,

$$\omega^2 = \frac{1}{r^3} \frac{\partial M_a^2}{\partial r}. \quad (31)$$

ROSSBY RADIUS OF DEFORMATION

- A fundamental horizontal length scale for a disturbance in a rotating fluid is the *Rossby radius of deformation*.
- The Rossby radius of deformation is the distance that a gravity wave (which is the means by which the fluid adjusts to equilibrium) will travel in one inertial period (ω^{-1}),

$$\lambda_R = c_g \tau_f = c_g / \omega. \quad (32)$$

where c_g is the speed of an inertial gravity wave.

- The Rossby radius of deformation is therefore

$$\lambda_R = \frac{c_g}{\sqrt{\eta(f_0 + 2\nu/r)}}. \quad (33)$$

- Note: For flows whose absolute vorticity is primarily due to planetary vorticity (i.e., flows where $\zeta \ll f$), (33) becomes

$$\lambda_R = c_g / f_0, \quad (34)$$

which is the form of the Rossby radius of deformation most commonly used in dynamic meteorology textbooks. However, (34) is just a less general form of (33).

- The gravity wave group velocity, c_g , depends on the stratification of the fluid.
 - For a barotropic fluid, it is given by $c_g = \sqrt{gH}$ where H is the depth of the fluid.

- For a baroclinic fluid, there are multiple *baroclinic modes* of oscillation, each with its own group velocity. For waves that travel primarily horizontally (long wavelengths) the group velocity can be approximated as

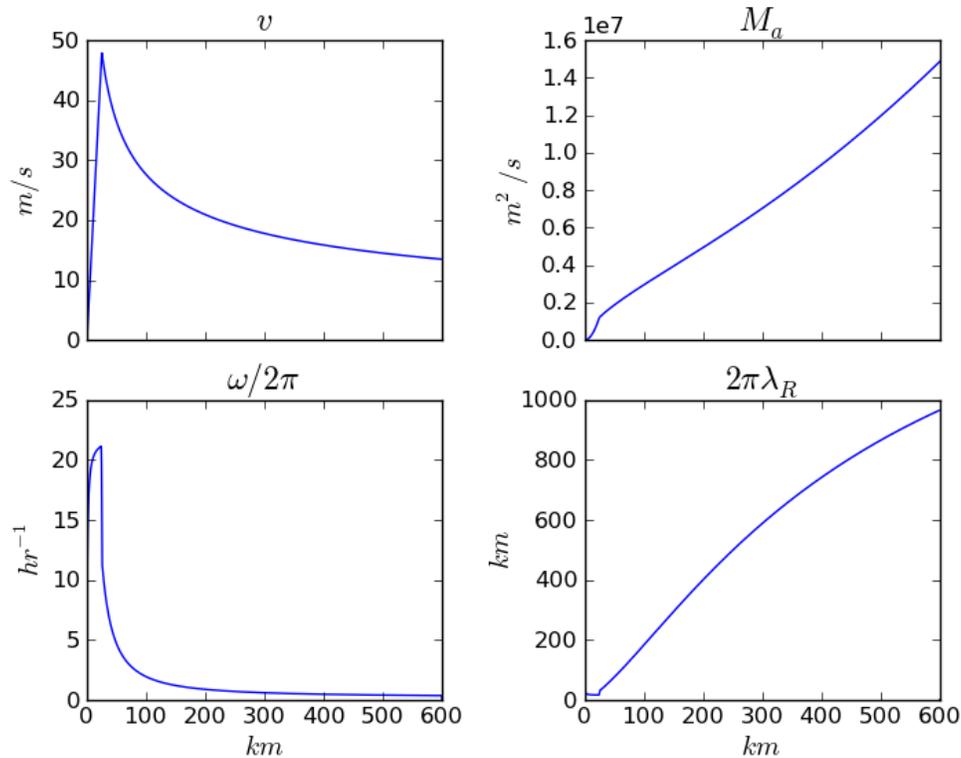
$$c_g \cong NH/n\pi. \quad (35)$$

where N is the Brunt-Vaisala frequency, H is the scale height of the atmosphere, and n refers to the different baroclinic modes of oscillation ($n = 1, 2, 3, \dots$).

- The baroclinic modes have a significantly slower group velocity than does the barotropic mode, and so the Rossby radius of deformation is smaller for the baroclinic modes.
- The response of the fluid to a disturbance can be assessed by comparing the horizontal length scale of the disturbance, L , to the Rossby radius of deformation.
 - For disturbances whose horizontal length scale, L , is much less than $2\pi\lambda_R$, the mass field will adjust to the velocity field.
 - For disturbances whose horizontal length scale, L , is much greater than $2\pi\lambda_R$, the velocity field will adjust to the mass field.
 - For disturbances in the intermediate range, mutual adjustment occurs.
- The size of the disturbance compared to the Rossby radius of deformation can also give us an idea of how persistent a circulation can be.
 - For disturbances whose size is comparable or larger than the Rossby radius of deformation, we know that the wind will adjust to the mass field, while for small disturbances the mass will adjust to the wind field.
 - Diabatic heating will cause the mass field to be perturbed.
 - If the heating is confined to a region much smaller than the Rossby radius of deformation, the disturbance in the mass field will not influence the wind.
 - All that will happen is that gravity waves will propagate away from the disturbance, and eventually nothing will be left of it.
 - Only if the heating is over a region comparable to the Rossby radius of deformation will a circulation in the wind field develop in response.
- Disturbances whose sizes are much less than the Rossby radius of deformation tend to die out quickly, and not develop a persistent circulation.

STRUCTURE AND DYNAMICS OF A TROPICAL CYCLONE

- The diagrams below show the wind speed, angular momentum, inertial frequency, and Rossby radius of deformation ($c_g = 10$ m/s) for a modified Rankin vortex at a latitude of 15° , with a radius of maximum winds of 25 km, an intensity of 48 m/s and $a = 0.5$.



- In particular, note that the Rossby radius of deformation is very large in the outer region of the cyclone, whereas it is much smaller in the core.
- Also notice the difference in the inertial frequency, with large values indicating high inertial stability in the core region, as compared with the outer region.

STRUCTURE OF THE CORE REGION

- The core region is 3 to 6 times the radius of maximum winds.
- The Rossby number (not shown in graphs above) is large in the core region, so the balance is cyclostrophic.
- The wind structure of the core has the following general characteristics:

- **Radius of maximum winds is generally in the eye wall.**
- **There is a sharp increase in speed near the radius of maximum winds.**
- **Winds decrease aloft.**
- **Maximum winds tilt outward with height.**
- **Maximum winds are found in the front-right quadrant of the cyclone.**
 - **This is true, *even if the forward speed of the storm is factored out.***
 - **The reason isn't completely understood.**
- **Strongest horizontal temperature gradients are near the radius of maximum winds.**
- **Warm core is narrow in the lower troposphere, and bulges outward aloft.**
- **Some features of the core region are the eye and rainbands.**
- **Some definitions:**
 - ***Rainband* – Also called *spiral bands*, or *feeder bands*, they are areas of cloud and of precipitation spiraling in toward the center of the circulation.**
 - ***Convective ring* – Rainbands that near completely circle the center.**
 - ***Moat* – Clear region between two convective rings.**
 - ***Eyewall* – Innermost convective ring.**
 - ***Eye* – Clear region at center of circulation, surrounded by the eyewall.**

DYNAMICS OF THE CORE REGION

- **The inner core region has very high inertial stability due to the large absolute vorticity and small radius.**
 - **This means it doesn't readily interact with its environment, and is why the inner core tends to be very symmetric.**
- **The inner core regions also has a small Rossby radius of deformation.**
 - **In the core, the wind tends to adjust to changes in the mass field. Any changes in the mass field due to heating/cooling or convergence/divergence will result in changes to the wind.**
 - **The intensity of the cyclone can therefore react rapidly to fluctuations in diabatic and latent heating, as well as fluctuations in vertically integrated divergence.**

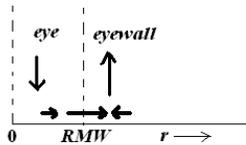
- The dynamics and fluctuations in the core are dominated by primarily by convection and heating.

THE EYE

- An eye generally doesn't form until the cyclone reaches hurricane or typhoon intensity.
- The mechanisms by which an eye is formed and maintained are not completely known. One conceptual model is presented below.
 - As air spirals inward toward the center of the cyclone, its speed will increase due to conservation of angular momentum.
 - However, if angular momentum were truly conserved, wind speeds of 600 m/s or more would be reached.
 - Therefore, angular momentum must be dissipated via convection, turbulence, and other processes that retard the flow.
 - The angular momentum balance at a point is given by

$$\frac{\partial M_a}{\partial t} = -u \frac{\partial M_a}{\partial r} - w \frac{\partial M_a}{\partial z} - F_r. \quad (36)$$

- The three terms on the right-hand-side are radial advection, vertical advection, and dissipation of angular momentum.
 - At some distance near the center of the vortex, the dissipation of angular momentum can no longer keep up with the horizontal advection. This leads to a horizontal convergence of angular momentum.
 - This horizontal convergence of angular momentum leads to an increase in the winds above the value that the pressure gradient can support (i.e., they are *super-gradient*).
 - The super-gradient winds develop a radially outward component, since the pressure gradient cannot supply the required centripetal acceleration. The consequences of this are (see figure):
 - There is radial convergence at distances outside of the radius of maximum winds.
 - There is radial divergence at distances inside the radius of maximum winds.



- The convergence leads to upward motion, and results in the convection within the eye wall.
 - The resultant convection within the eye wall serves to balance the horizontal convergence of angular momentum through vertical advection.
- The upward motion in the eye wall results in outflow aloft, and high perturbation pressure over the center of the storm.
- The upper-level pressure perturbation, combined with the low-level divergence within the eye itself, results in compensating subsidence in the eye.
- Due to the strong inertial stability in the eye wall, the strongest compensating subsidence is found on the inside of the radius of maximum winds.
- This results in an abrupt clearing just inside the eye wall.
- As the air subsides, the resultant compressional warming actually works against subsidence (through buoyancy).
- All that is needed in order to keep the eye relatively clear is enough subsidence to balance the buoyancy.
 - Thus, in the steady state, there doesn't have to be strong subsidence in the eye in order to maintain the eye.
- The maintenance of the eyewall is a balance between the horizontal and vertical advection of angular momentum. The radius of the eye can expand or contract depending on this balance.

EYE WALL REPLACEMENT CYCLE

- Sometimes an outer convective ring will form and will often cause the inner ring (the eye wall) to dissipate for two reasons.
 - The subsidence from the outer ring suppresses convection in the inner ring.
 - The outer ring robs the inner ring of inflow.

- Eye wall replacement usually results in a lowering of the intensity, since the new eye wall is at a larger radius, and from angular momentum arguments, would have a lower wind speed.
 - Intensity may build again once the new eye wall contracts.
- The causes and dynamics of eye wall replacement cycles are not well known.
- Eye wall replacement tends to occur in very intense cyclones.

RAINBANDS

- Rain bands are bands of convection that occur in the core region, and also can extend into the outer region of the cyclone.
- Rain bands can be tens of kilometers wide, and hundreds of kilometers in length.
- Convection often forms on the inside of the rain bands and moves up and forward through them.
- The region between the rain bands is characterized by subsidence.
- Rain bands may be of three types:
 - *Moving spiral bands* – These bands appear to rotate with the circulation.
 - *Convective rings* – A rain band that completely encloses the cyclone.
 - A convective ring may transition into an eye wall
 - *Principle spiral bands* – A non-moving rain band that wraps into the cyclone on the east side of the cyclone, and feeds lower latitude air into the vortex.
 - In a rough sense, principle spiral bands separate the inner region of the cyclone from the outer regions.
 - In the inner region, air makes several circuits as it spirals into the core, where in the outer region the air doesn't make it to the core.

STRUCTURE OF OUTER REGION

- In the outer region, the absolute vorticity is weaker and the radius is larger, so this region has much lower inertial stability than does the core region.
 - This means the outer region is less symmetric, and also more readily influenced by the environmental flow.
- The outer region also has a larger Rossby radius of deformation than does the core.

- In the outer region the mass field adjusts to the wind field.
- The tangential circulation can extend 1000 – 2000 km from the center of the storm.
- The radial circulation is much smaller in scale (~600 km).
- There is usually subsidence in the outer region, with areas of weak upward motion.
- A typical ring-like area of subsidence is found around 5° latitude or so from the center, and is sometimes referred to as the *moat*, since it results in clearing.

OUTFLOW REGION

- The intense updrafts in the eye wall carry cyclonic vorticity upwards. This results in the upper-level outflow near the cyclone center having cyclonic rotation.
- As it spreads outward, it loses positive vorticity and gains negative vorticity, so that it becomes anticyclonic as it moves away.
- The outflow region has weak inertial stability, and so doesn't resist horizontal flow.
 - The outflow region interacts readily with the environmental flow.
- The outflow mainly takes place in one or two *outflow jets* or *channels*.
 - These outflow channels are shallow, and are in the upper troposphere.
- Cyclones with two outflow channels tend to be more intense than those with a single outflow channel.

DIURNAL CYCLES

- Tropical cyclone convection shows a pronounced diurnal cycle.
- Convection is enhanced in the early morning hours (0300 to 0600 local), likely due to the same reasons that tropical convection over the open oceans has an early morning maximum.
- Areal coverage in cirrus clouds is maximum in the late afternoon (around 1800 local time).